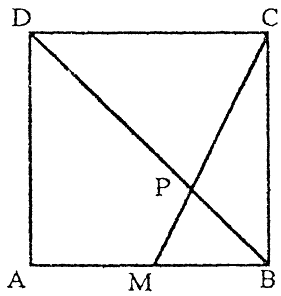
February 4, 1998 WOCOMAL VARSITY MEET

ROUND I: Similarity and Pythagorean Theorem

ALL ANSWERS MUST BE EXPRESSEED AS INTEGERS, REDUCED FRACTIONS, EXACT DECIMALS, OR IN SIMPLE RADICAL FORM. NO DECIMAL APPROXIMATIONS.

1. How big will an angle of $2\frac{1}{4}$ degrees seem if you look at it through a glass that magnifies an object 4 times? Answer in degrees.

2. 

M is the midpoint of side $\overline{AB}$ of square ABCD and AD = 1. Find length PB.

3. In triangle ABC, $a = 7$, $b = 8$, and $c = 9$. Find the length of the altitude to side c.

ANSWERS — Note the directions at the top of the page

(1 pt.) 1.____________________________

(2 pts) 2.____________________________

(3 pts) 3.____________________________

Auburn, Bromfield, Burncoat

ROUND II: Algebra I - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Karen is 10 and Kristy is 15. Karen knows that she will never catch up with Kristy in age, but she has heard that when one person is nine tenths as old as another, you can't really tell them apart. How long will it be until Karen is nine tenths as old as Kristy?

2. Determine the value of the product of x and y given that $3x + 2y = 7$ and $4x - 3y = 6.5$

3. Four numbers are written in a row. The average of the first two is 15. The average of the middle two is 8.7. The average of the last two is 24.6. What is the average of the first number and the last number?

ANSWERS

(1 pt.) 1.____________________ years

(2 pts) 2.____________________

(3 pts) 3.____________________

Algonquin, Auburn, Tantasqua

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ROUND III: Logarithms, exponents, radicals

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Express $\left(\log_3 3^{3a}\right)\left(\left(x^{-2}\right)^0\right)^{-1}\left(3+a^{-1}\right)^{-1}$ in terms of a.

2. $\sqrt[3]{4}\cdot\sqrt[4]{5}$ may be expressed in the form $\sqrt[12]{c}$. State the integral value of c.

3. In this problem, log means $\log_{10}$.

Let $\log 2 = a$ and $\log 3 = b$. Then $\log\frac{1}{6} + \log\frac{3}{10} + \log\frac{5}{14} + \ldots + \log\frac{13}{30}$

can be expressed in the form $ra + sb + t$, where r, s, and t are integers.
Find the value of $r + s + t$.

ANSWERS

(1 pt.) 1.__________________________

(2 pts) 2.__________________________

(3 pts) 3.__________________________

Assabet Valley, Holy Name, Hudson

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ROUND IV: Combinatorics

ALL ANSWERS MUST BE EXPRESSED AS SINGLE POSITIVE INTEGERS

1. There are 25 students in a class, but only 22 computers in the lab. How many different groups of 22 students are possible from the 25?

2. Eight dogs are running in a race. If three of them are clearly faster than the rest and sure to make up the top three finishers, in how many ~~ways can the race end~~? Assume no ties.
different orders can the race finish?

3. A set consists of the whole numbers 1 through 10. For how many different subsets does the sum of the elements in the subset equal 10? (Note, the order of elements in a set does not matter and no element may be repeated.)

ANSWERS
(1 pt.) 1.______________________

(2 pts) 2.______________________

(3 pts) 3.______________________

Mass. Academy, Shrewsbury, Tahanto

ROUND V: Analytic geometry of lines and conic sections

ALL ANSWERS MUST BE IN THE FORM SPECIFIED AND INVOLVE INTEGERS, REDUCED FRACTIONS, EXACT DECIMALS, OR SIMPLIFIED RADICALS. NO DECIMAL APPROXIMATIONS.

1. Write the $y = mx + b$ form equation of the line whose intercepts on the x and y axes are 5 and -3, respectively.

2. Point P(6, -2) lies on the circle with equation $x^2 - 4x + y^2 + 10y + 4 = 0$ Find the slope of the tangent line to the circle at P.

3. A parabola has focus (-4, 1) and directrix the y-axis. Find its equation in the form $x = ay^2 + by + c$

ANSWERS

(1 pt.) 1.________________________________

(2 pts) 2.________________________

(3 pts) 3._____________________________________

Algonquin, Hudson, Shepherd Hill

TEAM ROUND: Related Problem Solving

This whole round deals with Pythagorean triples. We consider only primitive triples (a, b, c) where a, b, and c are integers with no common factor greater than 1, $a < b < c$, and $a^2 + b^2 = c^2$. Within the sets to be described, the first triple has the smallest value of a, the second has the next smallest value of a, etc. When giving a triple as an answer, list it in the order a, b, c. All answers must be on the separate team answer sheet.

1. a, b, c, d. State, in order, the first four triples for which $c = b + 1$. (1 pt. each part)

2. a, b, c, d. State, in order, the first four triples for which $c = b + 2$. (1 pt. each part)

There are no Pythagorean triples of the form $(a, b, b + k)$ for $k = 3$ or 4.

3. Find (a) the smallest value of $k > 4$ for which $(a, b, b + k)$ may be a Pythagorean triple and (b) the first triple for that k

   (2 pts each part)

4. Find the first triple of the form $(a, b, b + 9)$ (3 pts)

5. Express the nth triple of the form $(a, b, b + 1)$ in terms of n only. (3 pts)

Norton Levy, QSC

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ROUND I

sim Pythag

1. 1 pt $2\frac{1}{4}°$ may omit degrees symbol

2. 2 pts $\frac{\sqrt{2}}{3}$ no decimal approx

3. 3 pts $\frac{8\sqrt{5}}{3}$

ROUND II

alg 1

1. 1 pt 35

2. 2 pts 1

3. 3 pts 30.9

ROUND III

logs exp rad

1. 1 pt $\frac{3a^2}{3a+1}$

2. 2 pts 32,000

3. 3 pts -8

ROUND IV

comb

1. 1 pt 2,300

2. 2 pts 720

3. 3 pts 10

ROUND V

analyt

1. 1 pt $y = \frac{3}{5}x - 3$

2. 2 pts $-\frac{4}{3}$

3. 3 pts $x = -\frac{1}{8}y^2 + \frac{1}{4}y - 2\frac{1}{8}$

exact decimals OK for each, 1. and 3. must have equations

TEAM ROUND pts vary

order does matter, throughout

1. (1 pt each)
a) 3, 4, 5
b) 5, 12, 13
c) 7, 24, 25
d) 9, 40, 41

2. (1 pt each)
a) 8, 15, 17
b) 12, 35, 37
c) 16, 63, 65
d) 20, 99, 101

3. (2 pts each)
a) 8
b) 20, 21, 29

4. 3 pts 33, 56, 65

5. 3 pts $2n+1,\ 2n^2+2n,\ 2n^2+2n+1$